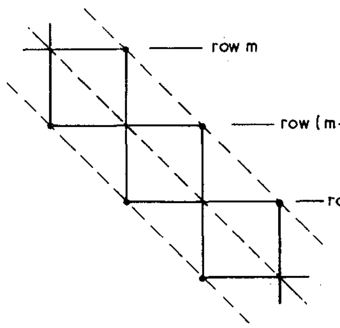

Fig. 2 Typical location of matrix coefficients determining the bandwidth at termination of the reduction.

The true minimum bandwidth is known to be 9 when a suitable sequence would be 1, 5, 9, 2, 6, 10, 3, 7, 11, 4, 8, 12.

Typical of this situation, for larger meshes, is the location of coefficients, which determine the bandwidth, on a square lattice as shown in Fig. 2. When rows m, $m + r$, $m + 2r$, etc. determine the bandwidth it is evident that a single interchange involving any of these rows will increase the bandwidth. For a similar mesh to the example but of size 5 by 10 instead of 2 by 3 and labeled in a similar manner, one such lattice would be at $m = 1$, $r = 12$.

Monotonic reduction of the bandwidth may be achieved by a series of simultaneous multiple cyclic interchanges involving many rows and columns. The procedure for finding cyclic interchanges is lengthy, and two alternatives are proposed.

Convergence either to the correct or pseudo minimum is dependent on the initial sequence. Thus, by starting each time with a random sequence and performing several reduction runs using the single interchange technique, if sufficient runs are performed one could give the true minimum. This proves simple but is unreliable.

A more rational alternative is to consider all possible sequences, but to discard blocks of sequences which are invalid at an early stage. Such an algorithm is described by Alway and Martin,[2] who also give an insight into the constraints imposed by the location of the coefficients which lead to the failure of the single interchange technique.

References

[1] Akyuz, F. A. and Utku, S., "An Automatic Node-Relabeling Scheme for Bandwidth Minimization of Stiffness Matrices," *AIAA Journal*, Vol. 6, No. 4, April 1968, pp. 728–730.

[2] Alway, G. G. and Martin, D. W., "An Algorithm for Reducing the Bandwidth of a Matrix of Symmetrical Configuration," *The Computer Journal*, Vol. 8, No. 3, Oct. 1965, pp. 264–272.

Reply by Author to J. Barlow and C. G. Marples

Fevzican A. Akyuz*
Jet Propulsion Laboratory, Pasadena, Calif.

THE conception of a theoretically failure-proof algorithm for the bandwidth minimization of the stiffness matrices would be a very simple task for a programmer. In fact, if n is the number of nodes in a given configuration, the survey of the $n!$ possible permutations for the node labels would yield the absolute minimum bandwidth. But, another survey of the number expressed by $n!$ shows that this approach is practically bound to a complete failure with increasing n within the ranges of the actual problem sizes; i.e., for $n = 15$, $n! \approx 10^{12}$, and the computation time would be of the order of 10^3 hr for such problems in the actual computers. Is it possible to devise a general algorithm, which, starting from any given configuration, would follow a reasonably smooth path to the absolute minimum bandwidth? After a considerable amount of computations with practical problems and test cases, using various criteria, the authors' conclusion is that the answer to this question might be affirmative, but yet the results might not have any practical value if the computer time increases drastically. Therefore, whether or not an algorithm might yield an absolute minimum bandwidth is not important. The criterion for success of an algorithm depends on its efficiency of obtaining some appreciable reduction of bandwidth within a reasonable computer time. Therefore, in any article on this topic, reference to the computer time must be the most essential factor. Reference 1, which was brought to our attention through Ref. 2, does indeed reflect these difficulties without any computer time data for basis of comparison.

With reference to Fig. 1a and 1b of Ref. 2, the inspection of the successive rows and columns shows that the algorithm of Ref. 3 fails without single interchange. This failure is due to the special form of the connectivity matrix for 3 × 2 division of the rectangular mesh, but it is hard to conceive any other mesh configuration of the plate or other topological form for which the scheme would fail. Actually, a suitable disturbance, i.e., isolation of a point by eliminating its connection with all adjacent points or an interchange of the labels of two arbitrary points, will restart the relabeling procedure leading generally to an optimum solution. An algorithm for automatic disturbance had been tried during the development of the actual program but has been discarded because of the increasing computer time. Usually the physical nature of the problem, i.e., boundary conditions in structural analysis, might introduce this type of disturbance.

The reasoning in paragraph 2 of Ref. 1 does not shadow the success of our basic algorithm. The inspections of the statements in the program list of ARAN[3] from 1280–1460 will show that there is a provision for temporary increase of bandwidth. The conditional statement following statement 1105 does not permit utilization of this provision because the failure of the program had not been detected in the test cases until the preparation date of Ref. 3. The program, as implemented in Ref. 4 late in 1967, utilizes this provision. The simple modification is shown in the first line of the errata list. To prove this point and to show the efficiency of the program, a series of test cases for rectangular plates of various mesh divisions, similar to that shown in Fig. 1a of Ref. 2, have been

Table 1 Results of test cases with various mesh divisions for rectangular plate

Mesh division	Number of nodes	Computation time t	Reduction of band area at time t, %	Reduction of band area to absolute minimum, %
10 × 5	66	10 sec[a]	30	35
20 × 10	231	100 sec	−10[b]	
		200 sec	13	
		234 sec[a]	31.5	39
34 × 14	525	100 sec	−4	
		5 min	−13	
		10 min	−13	
		15 min	−8	
		20 min	0.6	
		25 min	20.6	
		29 min[a]	37.8	46.8

[a] The last numbers in the time lists are the times at which the program stopped.
[b] The negative percentages refer to an increase of bandwidth.

Received September 3, 1968. Paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS 7-100, sponsored by NASA.

* Senior Research Engineer, Engineering Mechanics Division.

run on the IBM 7094, model I, and the results are summarized in Table 1.

Furthermore, in many problems, the total time for both relabeling and solution of the equations, where two or more unknowns were associated with each node, was less than the solution time without relabeling.

As a concluding remark, we have to note that single interchanges with repeated sweeping procedures are very promising. It is always possible to improve the existing provisions of the programs or implement various "look ahead" features without altering the basic philosophy of the scheme.

Errata†

The following errors and basic differences from the working program have been discovered in Ref. 3 in the listing of SUBROUTINE ARAN: 1) statement below 1105 should read IF (XSA-XSAP) 1107, 1107, 1108; 2) statement above 1290 should read IF (IJ-IN) 1290, 1290, 1310; 3) statement above 1600 should read CALL SEBIN (BCH, JBIP, NCH). In addition to these, the authors suggest that the empirical constant NCYCN = 3 + IN/100 be changed to NCYCN = IN to improve the chances of obtaining optimum bandwidth at the expense of increasing computation time.

References

[1] Alway, G. G. and Martin, D. W., "An Algorithm for Reducing the Bandwidth of a Matrix of Symmetrical Configuration," *The Computer Journal*, Vol. 8, 1965, pp. 264–272.

[2] Barlow, J. and Marples, C. G. "Comments on the Automatic Node-Relabeling Scheme for Bandwidth Minimization of Stiffness Matrices," *AIAA Journal*, Vol. 7, No. 2, Feb. 1969, pp. 380–381.

[3] Akyuz, F. A. and Utku, S., "An Automatic Node-Relabeling Scheme for Bandwidth Minimizations of Stiffness Matrices," *AIAA Journal*, Vol. 6, No. 4, April 1968, pp. 728–730.

[4] Utku, S. and Akyuz, F. A., "ELAS—A General Purpose Computer Program for the Equilibrium Problems of Linear Structures," TR 32-1240, *User's Manual*, Vol. 1, Jet Propulsion Lab., Feb. 1968.

† The author wishes to thank A. Whitney of McDonnell Douglas Corp. for pointing out the errors.

The Siren Revisited

William P. Rodden*
Los Angeles, Calif.

In a recent Note, Myklestad[1] recalls the glory that was once the desk calculator. He compares matrices to the Sirens. "Your next land-fall will be upon the Sirens: and these craze the wits of every mortal who gets so far. If a man come on them unwittingly and lend ear to their Siren-voices, he will never again behold wife and little ones rising to greet him with bright faces when he comes home from the sea." (Circe's warning to Odysseus, Ref. 2, p. 170.) This writer prefers to regard matrices as providing the means of leading Odysseus (the structural dynamicist and aeroelastician) past the Sirens (the Myklestad vibration analysis[3]?), between Scylla (the computer systems programmer) and Charybdis (the computer), and safely home to Penelope (the optimum structure).

The idealization of current aerospace structural designs as statically determinate systems of bending and twisting beam elements located along an elastic (elusive?) axis is an anachronism. The approximation may still be adequate for certain preliminary design purposes, but it can hardly be regarded as a reliable means to arrive at an optimum design for minimum weight with sufficient strength, stiffness, and life. Matrices of structural[4] and aerodynamic[5] influence coefficients do provide such a means. Beam methods may still continue to be useful for analysis of helicopter blades and long, slender missiles, but their utility in optimizing the design of highly swept or low-aspect ratio surfaces and large-diameter shell missile or fuselage structures for high-performance aerospace vehicles is gone with the wind—like the City of Troy.

Received August 19, 1968.
* Consulting Engineer. Associate Fellow AIAA.

References

[1] Myklestad, N. O., "The Matrix Siren," *AIAA Journal*, Vol. 5, No. 10, Oct. 1967, pp. 1897–1898.

[2] Shaw, T. E. (Lawrence of Arabia), *The Odessey of Homer*, Oxford University Press, London, 1956.

[3] Myklestad, N. O., *Fundamentals of Vibration Analysis*, McGraw-Hill, New York, 1956.

[4] Gallagher, R. H., *A Correlation Study of Methods of Matrix Structural Analysis*, AGARDograph 69, Pergamon Press, 1964.

[5] Rodden, W. P. and Revell, J. D., "Status of Unsteady Aerodynamic Influence Coefficients," Paper FF-33, 1962, IAS; preprinted as Rept. TDR-930(2230-09)TN-2, 1961, Aerospace Corp.

Reply by Author to W. P. Rodden

N. O. Myklestad*
The University of Texas at Arlington, Arlington, Texas

Mr. Rodden seems to have missed the point of my Note, since I was not comparing different methods of analyses at all. The gist of my message was that any method of analysis could be arranged in various ways for efficient numerical computations, and, if matrices were used for this purpose, it could result in greatly increased machine time. The real advantage of using matrices is in the simplication of the program, but, if the calculations are to be performed frequently, it may pay to write the program without the use of matrices.

Of course, if a matrix method of analysis is used the programming is automatically done in matrix form; but the method referred to in my Note was developed without the use of matrices, and later matrices were introduced ostensibly to make the method more efficient. However, the introduction of matrices in this case reduced the efficiency of the computer program by more than 50%.

Received September 27, 1968.
* Professor of Engineering Mechanics.

Comments on "Natural Frequencies of Clamped Cylindrical Shells"

Geoffrey B. Warburton,*
University of Nottingham,
Nottingham, United Kingdom

In a recent paper, Smith and Haft[1] used Flügge's equations,[2] as uncoupled by Yu,[3] to determine natural frequencies of clamped cylindrical shells. Nondimensional

Received July 22, 1968.
* Professor of Applied Mechanics.